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THE DOMINATION UNIFORM SUBDIVISION NUMBER OF $G \circ K_1$

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ABSTRACT. Let $G = (V, E)$ be an undirected and simple graph. A dominating set D of G is a set of vertices of G such that every vertex in $V - D$ is adjacent to at least one vertex in D. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality taken over all dominating sets of G. The domination uniform subdivision number of G is the least positive integer k such that the subdivision of any k edges from G results in a graph having domination number greater than that of G and is denoted by $usd_\gamma(G)$. In this paper, we discuss the domination uniform subdivision number for a standard graph operation namely corona of graphs.

1. INTRODUCTION

Let $G = (V, E)$ be a simple undirected graph of order n and size m. If $v \in V(G)$, then the neighborhood of v is the set $N(v)$ consisting of all vertices u which are adjacent to v. The closed neighborhood is $N[v] = N(v) \cup \{v\}$. The degree of v in G is $|N(v)|$ and is denoted by $deg(v)$. The maximum degree of G is $max\{deg(v) : v \in V(G)\}$ and is denoted by $\Delta(G)$. A vertex v is said to be full vertex if $deg(v) = n - 1$. A vertex v is said to be pendant vertex if $deg(v) = 1$. An edge incident with pendant vertex is called leaf or pendent edge. A path, a cycle and a complete graph on n vertices are denoted by P_n, C_n and K_n respectively.

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A complete bipartite graph is denoted by $K_{m,n}$. A graph is said to be connected if there exists a path between any pair of vertices. Otherwise it is said to be disconnected. A vertex is called support if it is adjacent to a pendant vertex. A support is said to be strong support if it is adjacent to more than one pendent. Let G be a connected graph. An edge $e = uv$ is said to be subdivided if it is deleted and replaced by a $u - v$ path of length two with a new internal vertex w (subdividing vertex). $G\Lambda\{e\}$ is the graph obtained by sub dividing the edge e. The corona of two graphs G_1 and G_2 is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2. Terms not defined here are used in the sense of [3].

A subset D of $V(G)$ is said to be a dominating set if every vertex of $V(G) - D$ is adjacent to at least one vertex in D. The minimum cardinality taken over all minimal dominating sets of G is the domination number of G and is denoted by $\gamma(G)$, see [4].

The domination subdivision number introduced by Arumugam Velammal in [5]. Its bound was obtained in [1] and several authors characterized trees according to their domination subdivision number. Also many results have also been obtained on the parameters $sd_{dd}, sd_{\gamma c}$ and $sd_{\gamma t}$. The domination subdivision number of a graph G is the minimum number of edges whose subdivision increases the domination number. It can also be defined as

$$sd_\gamma(G) = min\{|E^{'}| : \gamma(G\Lambda E^{'}) > \gamma(G)\} .$$

Here we have studied domination uniform subdivision number of $G \circ K_1$ for some standard graphs. In this paper the following theorems are used.

Theorem 1.1. *[2] For any graph G,* $0 \leq usd_\gamma(G \circ K_1) \leq m$. *Also the bounds are sharp.*

Theorem 1.2. *For any connected graph G,* $usd_\gamma(G \circ K_1) = m$ *if and only if* $G \cong K_{1,r} \circ K_1$ *, for all* $r \geq 1$.

2. Exact values for some standard graphs

In this section we attain domination uniform subdivision numbers of $P_n \circ K_1, C_n \circ K_1$ and $K_n \circ K_1$. Here, we consider corona of two graphs and in particular we take the second graph as K_1.

Definition 2.1. *[2]: The domination uniform subdivision number of G is the least positive integer k such that the subdivision of any k edges from G results in a graph having domination number greater than that of G and is denoted by $usd_\gamma(G)$. If it does not exist then $usd_\gamma(G) = 0$.*

Definition 2.2. *[2]: A subset $S \subseteq E(G)$ is said to be domination subdivision stable set if $\gamma(G\Lambda S) = \gamma(G)$. A domination subdivision stable set S is said to be maximum domination subdivision stable set if there is no stable subdivision set S' such that $|S'| > |S|$.*

Observation 2.1 ***[2]*** : $usd_\gamma(G) = |S| + 1$, where S is a maximum domination subdivision stable set of G.

Observation 2.2: Let $S \subseteq S'$. If S is not a domination subdivision stable set, then S' is also not a domination subdivision stable set.

Lemma 2.1. *Let G be a graph of order greater than 4. If there exists a path of length 3, $P^* = v_1e_1v_2e_2v_3e_3v_4$ where v_1 and v_4 are pendent vertices, then any set contains all the three edges of P^* is not a domination subdivision stable set of G.*

Proof. Any minimum dominating set of G must have any one of the pair from $\{(v_1, v_3), (v_1, v_4), (v_2, v_3), (v_2, v_4)\}$. Without loss of generality we assume that minimum dominating set D contains v_2 and v_3. Let G' be a graph obtained from G by subdividing all the edges of P^*. Let us take w_1, w_2 and w_3 be subdividing vertices. Then any minimum dominating set of G' should contain w_2 and w_3. Hence w_2 is not adjacent to any of the vertices of $(D\backslash\{v_1, v_2\}) \cup \{w_1, w_3\}$. Thus any minimum dominating set of G' contains w_2 and so $\gamma(G') > \gamma(G)$. That is $\gamma(G\Lambda\{e_1, e_2, e_3\}) > \gamma(G)$. Therefore $\{e_1, e_2, e_3\}$ is not a domination subdivision stable set. By Observation 2.2 , any set contains $\{e_1, e_2, e_3\}$ is not a domination subdivision stable set. $\square$

Theorem 2.1. *For any path $P_n (n \geq 2)$, $usd_\gamma(P_n \circ K_1) = \left\lceil \frac{3n}{2} \right\rceil$.*

Proof. Let $P_n = v_1e_1v_2e_2 \ . \ . \ . \ v_{n-1}e_{n-1}v_n$ be a path on n vertices and let $u_i^{'}s$ be pendent vertices and $x_j^{'}s$ be pendent edges of $P_n \circ K_1$ for all $i = 1, 2...n$. We can easily verify that the result is true for $n = 2$ and 3. Let us assume that $n \geq 4$.

Take $S = \{e_i/1 \leq i \leq n-1\} \cup \{x_j/j = 2k-1, 1 \leq k \leq \left\lceil \frac{n}{2} \right\rceil\}$.

Then $|S| = n - 1 + \leq \left\lceil \frac{n}{2} \right\rceil = \frac{2n-2}{2} + \left\lceil \frac{n}{2} \right\rceil = \left\lceil \frac{3n}{2} \right\rceil - 1$. By Lemma 2.1, S is a maximal domination subdivision stable set of $P_n \circ K_1$.

Claim: S is a maximum domination subdivision stable set of $P_n \circ K_1$. Suppose S is not a maximum domination subdivision stable set of $P_n \circ K_1$. Then there exists $S^{'} \subseteq E(P_n \circ K_1)$ which is a maximum domination subdivision stable set of $P_n \circ K_1$. Then $|S^{'}| > |S|$. Since S is a maximal domination subdivision stable set of $P_n \circ K_1$, $S \not\subset S^{'}$. Therefore $S \backslash S^{'}$ contains atleast one edge. Let it be e. Define $S^{''} \subseteq S^{'}$ such that $S^{''} = (S\backslash\{e\}) \cup \{x, y\}$, where $x, y \notin S$. Then there exists some P^* such that $S^{''}$ contains all the edges of P^*. Therefore by Lemma 2.1 $S^{''}$ is not a domination subdivision stable set and hence $S^{'}$ is not a domination subdivision stable set. Thus S is a maximum domination subdivision stable set. Hence $usd_\gamma(P_n \circ K_1) = |S| + 1 = \left\lceil \frac{3n}{2} \right\rceil$. □

Theorem 2.2. *For any cycle* $C_n, usd_\gamma(C_n \circ K_1) = \left\lfloor \frac{3n+2}{2} \right\rfloor$.

Proof. Let $C_n = v_1e_1v_2e_2 \ . \ . \ . \ v_ne_nv_1$ be a cycle of n vertices and let $u_i^{'}s$ be pendent vertices and $x_j^{'}s$ be pendent edges of $C_n \circ K_1$ for all $i = 1, 2 \ ...n$. We can easily verify that the result is true for $n = 3$. Let us assume that $n \geq 4$.

Take $S = \{e_i/1 \leq i \leq n\} \cup \{x_j/j = 2k-1, 1 \leq k \leq \left\lfloor \frac{n}{2} \right\rfloor\}$,

$|S| = n + \left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor \frac{2n+n}{2} \right\rfloor = \left\lfloor \frac{3n}{2} \right\rfloor$.

By Lemma 2.1, S is a maximal domination subdivision stable set of $C_n \circ K_1$.

Claim : S is a maximum domination subdivision stable set of $C_n \circ K_1$. Suppose S is not a maximum domination subdivision stable set of $C_n \circ K_1$. Then there exists $S^{'} \subseteq E(C_n \circ K_1)$ which is a maximum domination subdivision stable set of $C_n \circ K_1$. Then $|S^{'}| > |S|$. Since S is a maximal domination subdivision stable set of $C_n \circ K_1, S \not\subset S^{'}$. Therefore $S \backslash S^{'}$ contains atleast one edge. Let it be e. Define $S^{''} \subseteq S$ such that $S^{''} = (S\backslash\{e\}) \cup\{x, y\}$, where $x, y \notin S$. Then there exists some P^* such that $S^{''}$ contains all the edges of P^*. Therefore by Lemma 2.1, $S^{''}$ is not a domination subdivision stable set and hence $S^{'}$ is not a domination subdivision stable set. Thus S is a maximum domination subdivision stable set. Hence $usd_\gamma(C_n \circ K_1) = |S| + 1 = \left\lfloor \dfrac{3n+2}{2} \right\rfloor$. □

Theorem 2.3. *For any complete graph* $K_n, usd_\gamma(K_n \circ K_1) = \dfrac{n(n-1)}{2} + 2.$

Proof. Let $v_1, v_2 \ . \ . \ . \ v_n$ be vertices and $e_i (1 \leq i \leq nC_2)$ be edges of K_n respectively. Let $u_i{}'$s be pendent vertices and $x_i^{'}s$ be pendent edges of $K_n \circ K_1$ for all $i = 1, 2 \ . \ . \ . \ n$. Since K_n is complete there exists nC_2 edges in K_n and hence the graph $K_n \circ K_1$ consists of $nC_2 + n$ edges.

Let $S = \{e_i / 1 \leq i \leq nC_2\} \cup \{x_1\}$.

Then $|S| = nC_2 + 1$. By Lemma 2.1, S is a maximal domination subdivision stable set of $K_n \circ K_1$.

Claim : S is a maximum domination subdivision stable set of $K_n \circ K_1$. Suppose S is not a maximum domination subdivision stable set of $K_n \circ K_1$. Then there exists $S^{'} \subseteq E(K_n \circ K_1)$ which is a maximum domination subdivision stable set of $K_n \circ K_1$. Then $|S^{'}| > |S|$. Since S is a maximal domination subdivision stable set of $K_n \circ K_1$, $S \not\subset S^{'}$. Therefore $S \backslash S^{'}$ contains atleast one edge. Let it be e. Define $S^{''} \subseteq S^{'}$ such that $S^{''} = (S \backslash \{e\}) \cup \{x, y\}$, where $x, y \notin S$.

Then there exists some P^* such that $S^{''}$ contains all the edges of P^*. Therefore by Lemma 2.1 $S^{''}$ is not a domination subdivision stable set and hence $S^{'}$ is not a domination subdivision stable set. Thus S is a maximum domination subdivision stable set. Hence $usd_\gamma(K_n \circ K_1) = |S| + 1 = \dfrac{n(n-1)}{2} + 2.$ □

3. BOUNDS FOR $usd_\gamma(G \circ K_1)$

Theorem 3.1. *Let G be a connected graph,* $m + 2 \leq usd_\gamma(G \circ K_1) \leq m + n, n \geq 1.$

Proof. Let the edge set of $G \circ K_1$ consists of all the edges in G and all the edges joining K_1 to corresponding vertex of G. Therefore $|E(G \circ K_1)| = m + n$.

Hence $usd_\gamma(G \circ K_1) \leq m + n$. Since all the vertices of G are support vertices of $G \circ K_1, E(G)$ is a domination subdivision stable set of G. In particular $E(K_n) \cup \{e\}$, where e is a pendent vertex of $K_n \circ K_1$, is a maximum domination subdivision stable set of $K_n \circ K_1$ by Theorem 2.3. By Observation 2.1 $usd_\gamma(G) = S + 1$, where S is a maximum domination subdivision stable set. Therefore $usd_\gamma(K_n \circ K_1) = m + 2$. Then $usd_\gamma(G \circ K_1) \geq m + 2.$ □

Theorem 3.2. *For any connected graph G, $usd_\gamma(G \circ K_1) = m + n$ if and only if G is a star graph.*

Proof. The proof follows from Theorem 1.2. □

Theorem 3.3. *Let G be a connected graph of order n, $usd_\gamma(G \circ K_1) = m + 2$ if and only if $G \cong K_n$.*

Proof. Assume that $usd_\gamma(G \circ K_1) = m + 2$. Suppose $G \cong K_n$. Then there exists a pair of non-adjacent vertices, say e_1 and e_2. Therefore $E(G) \cup \{e_1, e_2\}$ be a domination subdivision stable set. Thus $usd_\gamma(G \circ K_1) > m + 2$. Conversely assume that $G \cong K_n$. Then by Theorem 2.3 $usd_\gamma(G \circ K_1) = m + 2$. □

CONCLUSION

In this paper, we obtain domination uniform subdivision numbers of corona product on K_1 and some standard graphs. Finally, we determine the bounds of $usd_\gamma(G \circ K_1)$ for any connected graph and characterize the extremal graphs of the bounds.

REFERENCES

[1] A. BHATTACHARYA, G. R. VIJAYAKUMAR: *Effect of Edge-Subdivision on Vertex-Domination in a Graph* , Discussiones Mathematicae Graph Theory, **22** (2002), 335–347.

[2] M. K. ANGEL JEBITHA: *Domination Uniform Subdivision Number of Graphs,* International Journal of Mathematics Trends and Technology, **27**(1) (2015), 1–6.

[3] G. CHARTRAND, P. ZHANG: *Introduction to Graph Theory*, Tata McGraw-Hill, 2006.

[4] T. W. HAYNES, S. T. HEDETNIEMI, P. J. SLATER: *Fundamentals of Domination in Graphs* , Marcel Dekker, 1998.

[5] S. VELAMMAL: *Studies in Graph Theory* , Covering, Independence, Domination and Related Topics, Ph. D. Thesis, 1997.

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